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Composite power law holdup correlations in horizontal pipes

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Abstract

A wide range of experimental holdup data, from different sources, are analyzed based on a theoretical model proposed in this work to evaluate the holdup in horizontal pipes. 2276 gas–liquid flow experiments in horizontal pipelines with a wide range of operational conditions and fluid properties are included in the database. The experiments are classified by mixture Reynolds number ranges and composite analytical expressions for the relationship between the liquid holdup and no-slip liquid holdup vs. the gas–liquid volumetric flow rate are obtained by fitting the data with logistic dose curves. The Reynolds number appropriate to classify the experimental data for gas–liquid flows in horizontal pipes is based on the mixture velocity and the liquid kinematic viscosity. Composite power law holdup correlations for flows sorted by flow pattern are also obtained. Error estimates for the predicted vs. measured holdup correlations together with standard deviation for each correlation are presented. The accuracy of the correlations developed in this study is compared with the accuracy of 26 previous correlations and models in the literature. Our correlations predict the liquid holdup in horizontal pipes with much greater accuracy than those presented by previous authors.

Keywords: Holdup; Gas–liquid; Power law; Pipe flow; Flow type

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1. Introduction

This study deals with the prediction of liquid holdup in horizontal pipes. This problem is of great interest in many industries, especially in the oil industry in which the estimate of the production is intimately related with this parameter.

Numerous efforts have been made in to develop methods to determine liquid holdup in pipes. Empirical and semiempirical correlations based on experimental data and diverse theoretical models with different degrees of complexity for predicting liquid holdup in pipes have been suggested by a number of investigators. Some authors have attempted to find general correlations for liquid holdup in two-phase flow by curve fitting of experimental data (e.g. Lockhart and Martinelli, 1949; Beggs and Brill, 1973; Abdul-Majeed, 1996). Other authors have developed models and correlations specific for each flow type: Stratified flow (e.g. Agrawal, 1971; Chen and Spedding, 1981), Annular flow (e.g. Kadambi, 1985; Tandon et al., 1985) and slug flow (e.g. Bonnecaze et al., 1971; Mattar and Gregory, 1974; Gregory et al., 1978; Gómez et al., 2000).

The correlations are strongly dependent on the composition of the database used. Most of these correlations and models have been developed and/or validated using experimental data with a limited range of operational conditions.

In this work, data from 2276 experiments for a wide range of operational conditions and fluid properties taken from different sources, are analyzed using a theoretical model proposed to evaluate the holdup in horizontal pipes. The theoretical model relates the liquid holdup and no-slip liquid holdup (H_L/λ_L) to the gas–liquid volumetric flow rates relation (Q_G/Q_L) for different mixture Reynolds numbers ranges. The Reynolds number appropriate for gas–liquid flows in horizontal pipes is based on the mixture velocity and the liquid kinematic viscosity. This Reynolds number is same to that defined by García et al. (2003). The proposed theoretical model fits well the experimental data for values of $Q_G/Q_L < 10$ and $Re < 300\,000$. However, for values of $Q_G/Q_L < 1$ and $Re \geqslant 300\,000$ and $Q_G/Q_L > 10$ for all Reynolds numbers, the experimental values distributions of H_L/λ_L vs. Q_G/Q_L and the distributions obtained applying the proposed theoretical model are significantly different.

When the experimental values of H_L/λ_L vs. Q_G/Q_L for different intervals of Reynolds numbers are plotted, typical composite power laws formed using logistic dose curves are obtained (Patankar et al., 2002). Excellent results in the study of two-phase flow problems have been obtained by adjusting the experimental data using this kind of curves (Joseph, 2002; Patankar et al., 2001a,b, 2002; Wang et al., 2002; Pan et al., 2002; Viana et al., 2003; Mata et al., submitted for publication; García et al., 2003).

In this study we obtained a group of composite power law correlations of H_L/λ_L vs. Q_G/Q_L for different intervals of Reynolds numbers fitting 2276 experimental data without sorting according to flow type.

It is well known that the liquid holdup depends on the flow type. The experimental data were processed and sorted according to flow type and we obtained a group of composite power law correlations of H_L/λ_L vs. Q_G/Q_L for different intervals of Reynolds numbers for each flow type. Of course, the correlations for separate flow patterns are more accurate but possibly less useful than those for which previous knowledge of actual flow pattern is not required. A correlation for which a flow pattern is not specified is exactly what is needed in a field situation in which the flow pattern is unknown.

The accuracy of the correlations developed in this paper is evaluated in two ways; by comparing predictions with the data from which the correlations are derived and by comparing the predictions of our correlations with the predictions of models and correlations of other authors. We compared our predictions of holdup with those obtained from the correlations of Armand (1946), Lockhart and Martinelli (1949), Flanigan (1958), Hoogendoorn (1959), Levy (1960), Hughmark (1962), Zivi (1963), Guzhov et al. (1967), Eaton et al. (1967), Bonnecaze et al. (1971), Beggs and Brill (1973), Mattar and Gregory (1974), Butterworth (1975), Gregory et al. (1978), Chen and Spedding (1981), Chen and Spedding (1983), Spedding and Chen (1984), Minami and Brill (1987), Hart et al. (1989), Spedding et al. (1998) and Gómez et al. (2000) as well as with the predictions of the homogeneous flow model and with the predictions of the models of Nishino and Yamazaki (1963), Turner and Wallis (1965) and Tandon et al. (1985), Abdul-Majeed (1996). For completeness and comparison purposes these correlations and models are presented in Appendix A.

Statistical parameters to evaluate the prediction of the liquid holdup of 2276 experimental data by our correlations and of the correlations and models in the literature were determined. The comparison of the holdup prediction of the models was carried out using the average absolute percent error. In general, our correlations predict the holdup with much greater accuracy than those presented by previous authors.

2. Theoretical model

Butterworth (1975) by intuitive reasoning proposed that the more commonly used holdup prediction equations may be represented by the relation,

$$\frac{H_L}{H_G} = A\left[\frac{1-x}{x}\right]^p \left[\frac{\rho_G}{\rho_L}\right]^q \left[\frac{\mu_L}{\mu_G}\right]^r \tag{1}$$

where H is the holdup, x the dryness fraction, ρ the density and μ the dynamic viscosity. The subscripts L and G refer to the liquid phase and gas phase, respectively. A, p, q and r are dependent parameters of flow pattern considered.

Later, Chen and Spedding (1983) justified the Butterworth (1975) correlation for certain conditions and expressed this equations in terms of the volumetric flow rate, Q, instead of dryness fraction,

$$\frac{1}{H_L} = 1 + K\left[\frac{Q_G}{Q_L}\right]^a \left[\frac{\rho_G}{\rho_L}\right]^b \left[\frac{\mu_G}{\mu_L}\right]^c \tag{2}$$

In this work, combining eq. (2) with the no-slip holdup λ_L equation, a new relation between holdup, no-slip holdup and Q_G/Q_L is proposed:

$$\frac{H_L}{\lambda_L} = \frac{1 + (Q_G/Q_L)}{1 + C(Q_G/Q_L)^a} \tag{3}$$

where a and $C = K[\frac{\rho_G}{\rho_L}]^b[\frac{\mu_G}{\mu_L}]^c$ are parameters that depends on the fluid properties and the flow pattern.

In order to obtain a and C, we use gas flow rate Q_G, liquid flow rate Q_L and holdup measurements H_L corresponding to 2276 experimental points taken from Intevep's databank, the Stanford multiphase flow database (SMFD), the database of the Tulsa University fluid flow projects (TUFFP) and other literature sources for gas–liquid flow in horizontal pipes. These data are summarized Tables 1–4. The columns in the tables are self explanatory except that "No Exp" means the number of experiments, ε/D is the average size of pipe wall roughness over pipe diameter, FP means "flow pattern" and AN, DB, SL, SS and SW stand for annular, dispersed bubble, slug, stratified smooth and stratified wavy flow, respectively.

According to the literature review, this database gathers the widest range of operational conditions and fluid properties so far compiled for holdup correlations: $1 \leqslant \mu_L \leqslant 1200$ cP, $0.01 \leqslant Q_G/Q_L \leqslant 33493$, $0.002 \leqslant H_L \leqslant 0.99$, $0.0232 \leqslant D \leqslant 0.1402$ m, $0 \leqslant \varepsilon/D \leqslant 1.710^{-3}$.

Fig. 1 shows the H_L/λ_L relation against Q_G/Q_L for the entire database (2276 experimental points).

The scatter shown in Fig. 1 could be significantly reduced if the experimental data are classified in ranges of mixture Reynolds numbers defined as:

$$Re = \frac{U_M D}{\nu_L} \tag{4}$$

where $\nu_L = \mu_L/\rho_L$ is the kinematic viscosity of the liquid. This Reynolds number definition is similar to that used by García et al. (2003) to correlate the Fanning friction factor for laminar and turbulent gas–liquid flow in horizontal pipelines.

In this work, reasonably good correlations are obtained fitting the data with the theoretical model defined by Eq. (3) for eight different ranges of mixture Reynolds numbers. The parameters a and C for this correlations were obtained fitting Eq. (3) to the 2276 data points using the non linear optimization method of Microsoft® Excel Solver minimizing the residual mean square, and are presented in Table 5.

The theoretical model correlations (TMC) for each Reynolds number range are shown in Figs. 2 and 3.

In order to compare predicted liquid holdup $(H_L)_{pred}$ with experimental data $(H_L)_{expe}$, we use the following eight commonly used statistical parameters (Gregory and Fogarasi, 1985; Xiao

Table 1
Intevep date

Source	No. exp	Fluids	μ_L [cP]	Q_G/Q_L	H_L	D [m]	ε/D	FP
Rivero et al. (1995)	74	Air–water Air–oil	1–200	7–442	0.05–0.34	0.0508	0	SW
Ortega et al. (2000)	23	Air–oil	500	0.38–49	0.62–0.99	0.0508	0	SL
Cabello et al. (2001)	17	Air–kerosene	1	0.46–26	0.67–0.95	0.0508	0	SL
Ortega et al. (2001)	24	Air–oil	1200	0.57–18	0.75–0.97	0.0508	0	SL
Mata et al. (submitted for publication)	22	Air–oil	100	0.12–31	0.41–0.98	0.0254	0	SL
Dos Santos (2002)	22	Air–oil	100	0.01–11	0.49–0.99	0.0254	0	DB SL

Table 2
Sanford data

Source	No. exp	Fluids	μ_L [cP]	Q_G/Q_L	H_L	D [m]	ε/D	FP
Alves (1954)	27	Air–oil	80	0.10–556	0.12–0.93	0.0266	1.7×10^{-3}	AN SL SW
Govier and Omer (1962)	56	Air–water	1	0.07–1702	0.03–0.96	0.0261	0	AN SL SS SW
Eaton (1966)	51	Gas–water	1	0.58–432	0.01–0.73	0.0508	8.0×10^{-4}	SL SS SW
Agrawal (1971)	19	Air–oil	5	3.21–440	0.15–0.85	0.0258	0	SS
Yu (1972)	15	Air–oil	5	0.23–6	0.41–0.89	0.0258	0	SL
Mattar (1973)	8	Air–oil	5	0.32–26	0.24–0.77	0.0258	0	SL
Aziz et al. (1974)	120	Air–oil	5	0.01–243	0.33–0.99	0.0258	0	DB SL
Companies[a]	139	Air–HL	3–19	0.08–578	0.05–0.99	0.0232	6.5×10^{-5}	AN
	144	Air–water	3–19	0.07–547	0.05–0.99	0.0237	6.5×10^{-5}	SS
	54	Air–oil	1–25	0.07–222	0.02–0.99	0.0381	1.2×10^{-3}	SL
	208		1	0.08–22092	0.004–0.95	0.0455	0	SW
	442		3–15	0.02–1561	0.01–0.99	0.0502	3.0×10^{-5}	
	117		3–22	0.04–1398	0.01–0.97	0.0909	1.7×10^{-5}	
	151		3–20	0.02–197	0.07–0.99	0.1402	1.1×10^{-5}	

HL: Hydrocarbon liquid.
[a] Data set are identified as: SU28, SU29, SU184-187, SU199, SU24, SU25 and SU26.

et al., 1990; Ouyang, 1995; García et al., 2003; García, 2004). The statistical parameters are defined as:

Average percent error E_1,

$$E_1 = \frac{1}{n}\sum_{i=1}^{n} r_i \tag{5}$$

Average absolute percent error E_2,

$$E_2 = \frac{1}{n}\sum_{i=1}^{n} |r_i| \tag{6}$$

Standard percent deviation E_3

$$E_3 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - E_1)^2} \tag{7}$$

Table 3
Tulsa data

Source	No. exp	μ_L [cP]	Q_G/Q_L	H_L	D [m]	ε/D	FP
Beggs (1972)	19 22	1	0.62–664 0.23–669	0.02–0.68 0.02–0.83	0.0254 0.0381	0	AN DB SL SS
Cheremisinoff (1977)	148	1	58–1000	0.01–0.20	0.0635	0	SS SW
Mukherjee (1979)	33	1	0.14–424	0.03–0.92	0.0381	3.0×10^{-5}	AN SL SS SW
Andritsos (1986)	90 14	1–70 80	130–33493 361–18922	0.002–0.36 0.003–0.21	0.0252 0.0953	0	AN SL SS SW
Kokal (1987)	10 13	8	19–384 13–179	0.05–0.37 0.09–0.35	0.0512 0.0763	0	SS SW

Table 4
Other sources

Source	No. exp	Fluids	μ_L [cP]	Q_G/Q_L	H_L	D [m]	ε/D	FP
Minami and Brill (1987)	57 54	Air–kerosene Air–water	1	7–2564 7–1737	0.01–0.44 0.01–0.45	0.0779	4.6×10^{-5}	AN SL SS SW
Abdul-Majeed (1996)	83	Air–kerosene	1	0.98–2534	0.01–0.61	0.0508	4.6×10^{-5}	AN SL SS SW

Root mean square percent error E_4,

$$E_4 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i)^2} \tag{8}$$

Average error E_5,

$$E_5 = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{9}$$

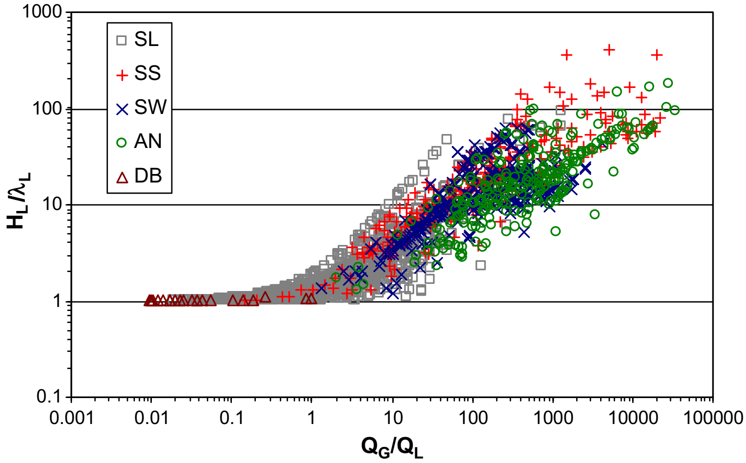

Fig. 1. H_L/λ_L relation against Q_G/Q_L for the entire database.

Table 5
Parameters a and C of theoretical model, Eq. (3)

Range	C	a
$Re < 2000$	0.3372	0.6390
$2000 \leqslant Re < 5000$	0.4379	0.4583
$5000 \leqslant Re < 10\,000$	0.4424	0.5568
$10\,000 \leqslant Re < 20\,000$	0.5693	0.5147
$20\,000 \leqslant Re < 40\,000$	0.6215	0.5395
$40\,000 \leqslant Re < 100\,000$	0.7095	0.5673
$100\,000 \leqslant Re < 300\,000$	0.6735	0.6252
$300\,000 \leqslant Re < 2\,670\,000$	1.1916	0.5407

Average absolute error E_6,

$$E_6 = \frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{10}$$

Standard deviation E_7,

$$E_7 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e_i - E_5)^2} \tag{11}$$

Root mean square error E_8

$$E_8 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e_i)^2} \tag{12}$$

where, $r_i = [\frac{(H_L)_{pred}-(H_L)_{expe}}{(H_L)_{expe}}]100$, $e_i = (H_L)_{pred} - (H_L)_{expe}$ and n is the number of the experimental data.

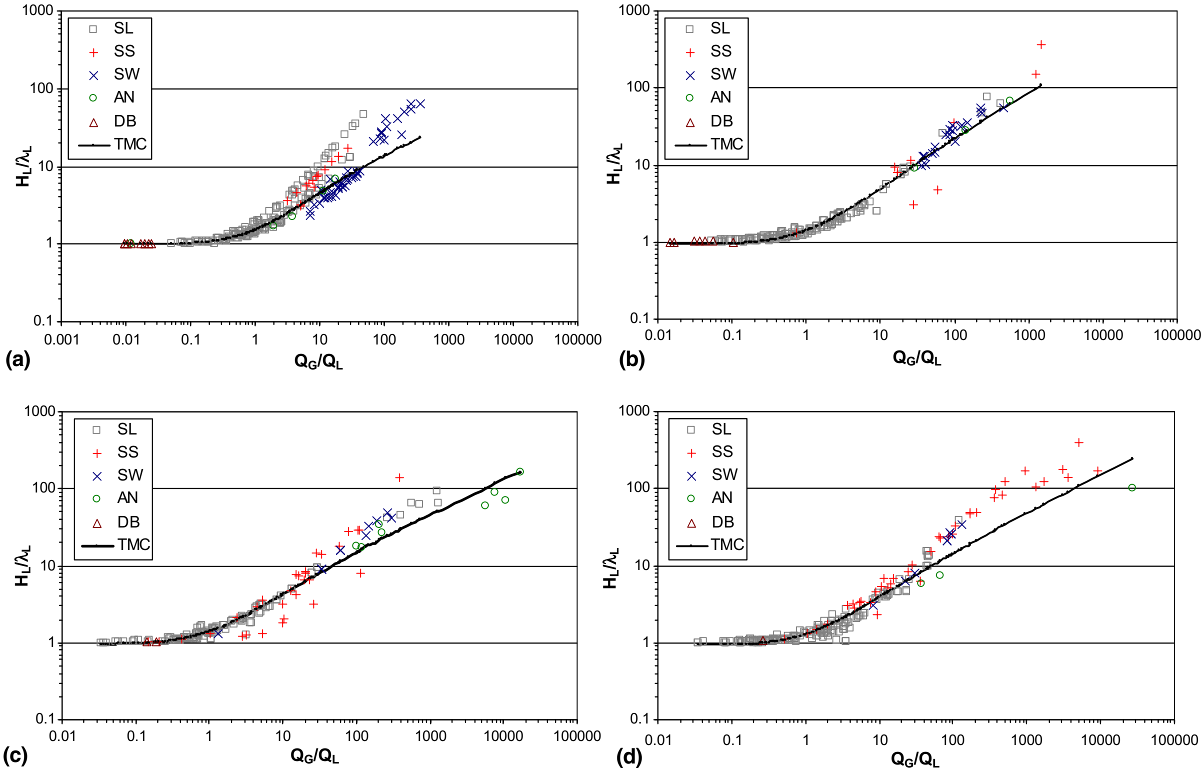

Fig. 2. Theoretical model correlations for (a) $Re < 2000$, (b) $2000 \leqslant Re < 5000$, (c) $5000 \leqslant Re < 10\,000$, (d)$10\,000 \leqslant Re < 20\,000$.

The average percent error E_1 is a measure of the agreement between predicted and measured data. It indicates the degree of overprediction (positive values) or underprediction (negative values). Similarly, the average absolute percent error E_2 is a measure of the agreement between predicted and measured data. However, in this parameter the positive errors and the negative errors do not cancel each other. For this reason, the average absolute percent error is considered a key parameter in order to evaluate the prediction capability of models and correlations. The standard deviation percent error E_3 indicates how large the errors are on the average. The root mean square percent error E_4 indicates how close the predictions are to the experimental data. The statistical parameters E_5, E_6, E_7 and E_8 are similar to E_1, E_2, E_3 and E_4 but the difference is that they are not based on the errors relative to the experimental liquid holdup.

The statistical parameters E_1–E_8 for theoretical model correlations are presented in Table 6.

The theoretical model correlations have an average error of −5.5% and an average absolute error of 24%. Only 68.7% of the points (1563 experimental points) are in the band between ±30%. The best agreements are obtained for slug and dispersed bubble flow data, with an average absolute error of 14.7% and 2.2%, respectively. The worst agreements are obtained for annular and stratified flow data, with an average absolute error of 35% and 36.4%, respectively.

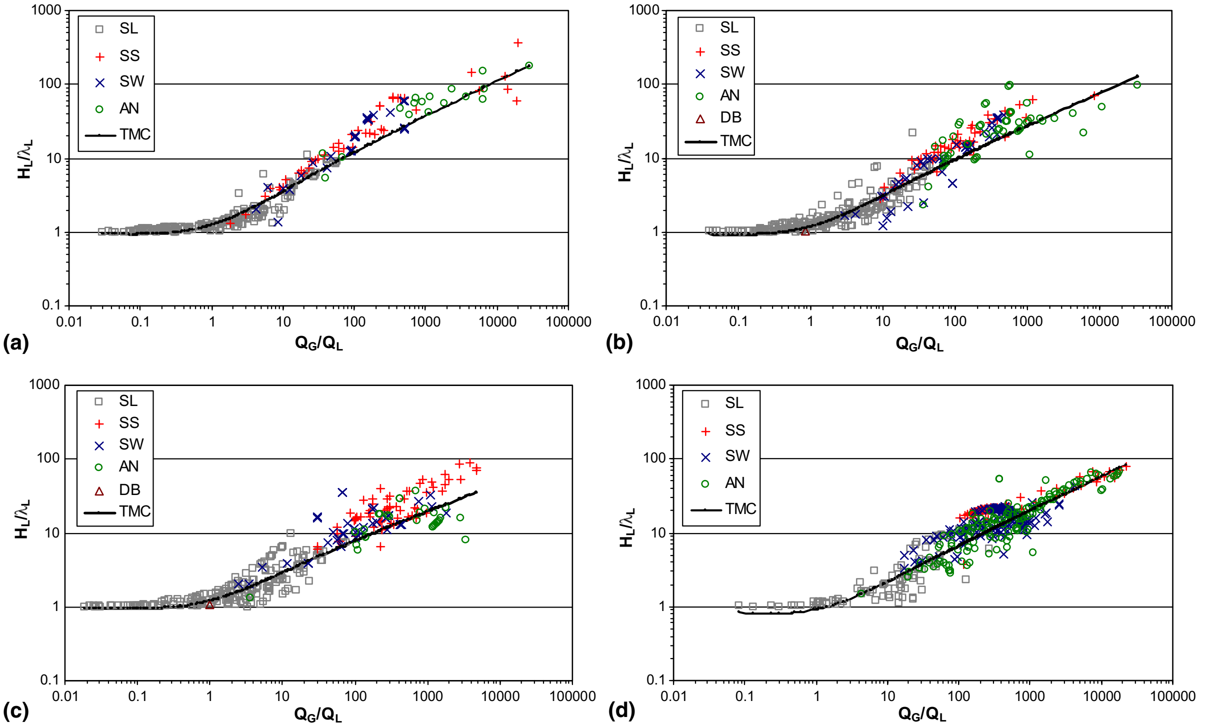

Fig. 3. Theoretical model correlations for (a) $20000 \leqslant Re < 40000$, (b) $40000 \leqslant Re < 100000$, (c) $100000 \leqslant Re < 300000$, (d) $Re \geqslant 300000$.

Table 6
Statistical parameters for theoretical model correlations

Range	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
$Re < 2000$	−10.65	18.53	25.17	27.34	−2.23	2.50	6.62	6.99
$2000 \leqslant Re < 5000$	0.01	14.64	29.16	29.16	−2.40	2.86	18.38	18.54
$5000 \leqslant Re < 10000$	0.41	17.87	31.07	31.07	−1.64	3.68	12.36	12.47
$10000 \leqslant Re < 20000$	−4.01	19.34	29.01	29.29	−5.05	6.71	27.38	27.84
$20000 \leqslant Re < 40000$	−4.78	21.20	30.32	30.69	−3.45	4.87	16.76	17.11
$40000 \leqslant Re < 100000$	−6.45	25.75	35.84	36.41	−2.48	3.68	8.81	9.16
$100000 \leqslant Re < 300000$	−8.60	29.46	39.13	40.07	−3.22	4.30	8.74	9.32
$300000 \leqslant Re < 2670000$	−8.20	34.50	44.18	44.93	−2.89	5.04	6.31	6.94

Analyzing the experimental data distribution in Figs. 2 and 3, it is evident that in general the theoretical model does not fit the data well for $Q_G/Q_L > 10$. On the other hand, for large values of Q_G/Q_L, the slope of the equation proposal should be positive. The zone between the extreme values is not clearly defined. An equation in which the H_L/λ_L relation approaches 1 for very small values of Q_G/Q_L is required. Consequently, particular composite power laws for each mixture Reynolds number range could be obtained fitting the experimental data with logistic dose response curves applying a technique described by Barree (Patankar et al., 2002).

3. Universal (all flow patterns) composite holdup correlations (UCHC)

Single equations (called composite) that can be used to predict H_L/λ_L for a wide range of gas and liquid flow rates, viscosity values and different flow patterns were obtained fitting data with logistic dose response curves for different mixture Reynolds number ranges. The particular composite power laws equation is given by

$$\frac{H_L}{\lambda_L} = F + \frac{(1-F)}{\left(1+\left(\frac{1}{t}\left(\frac{Q_G}{Q_L}\right)\right)^c\right)^d} \tag{13}$$

where F is a power law defined as

$$F = aRe^b \tag{14}$$

a, b, c, d and t are parameters obtained fitting Eq. (13) to the 2276 data points using the non linear optimization method of Microsoft® Excel Solver minimizing the residual mean square. The parameters a, b, c, d and t for this correlation are presented in Table 7.

The universal composite correlations (UCC) for each Reynolds number range are shown in Figs. 4 and 5.

The average percent error E_1, the average absolute percent error E_2, the standard percent deviation E_3, the root mean square percent error E_4, the average error E_5, the average absolute error E_6, the standard deviation E_7, the root mean square error E_8 for each correlation are presented in Table 8.

The universal holdup correlations have an average error of −4.1% and an average absolute error of 21.0%. 73.9% of the points (1682 experimental points) are in the band between ±30%. The best agreements are obtained for slug and dispersed bubble flow data, with an average absolute error of 13.1% and 1.9%, respectively. The worst agreements are obtained for annular and stratified flow data, with an average absolute error of 34.9% and 31.3%, respectively.

4. Holdup correlations sorted by flow pattern (FPHC)

Each and every experiment was classified by flow type: 1305 slug flow, 20 dispersed bubble, 692 stratified flow and 259 annular flow. The experimental data for each flow pattern were classified

Table 7
Parameters of the universal composite correlations for holdup

Range	a	b	c	d	t
$Re < 2000$	85.5969	0.4503	0.4240	0.0781	432.0226
$2000 \leqslant Re < 5000$	73.9792	0.2936	0.6536	0.2634	429.8162
$5000 \leqslant Re < 10\,000$	74.1824	0.0001	0.9458	1.5020	430.7731
$10\,000 \leqslant Re < 20\,000$	70.5777	0.1238	1.04086	0.3322	107.5723
$20\,000 \leqslant Re < 40\,000$	70.5791	0.04267	1.0423	0.3410	107.5740
$40\,000 \leqslant Re < 100\,000$	17.5825	0.1077	0.8963	0.9592	151.007
$100\,000 \leqslant Re < 300\,000$	2.5383	0.3001	0.8655	3.5587	100.0044
$300\,000 \leqslant Re < 2\,670\,000$	1.4976	0.3820	0.9985	2.5626	99.9486

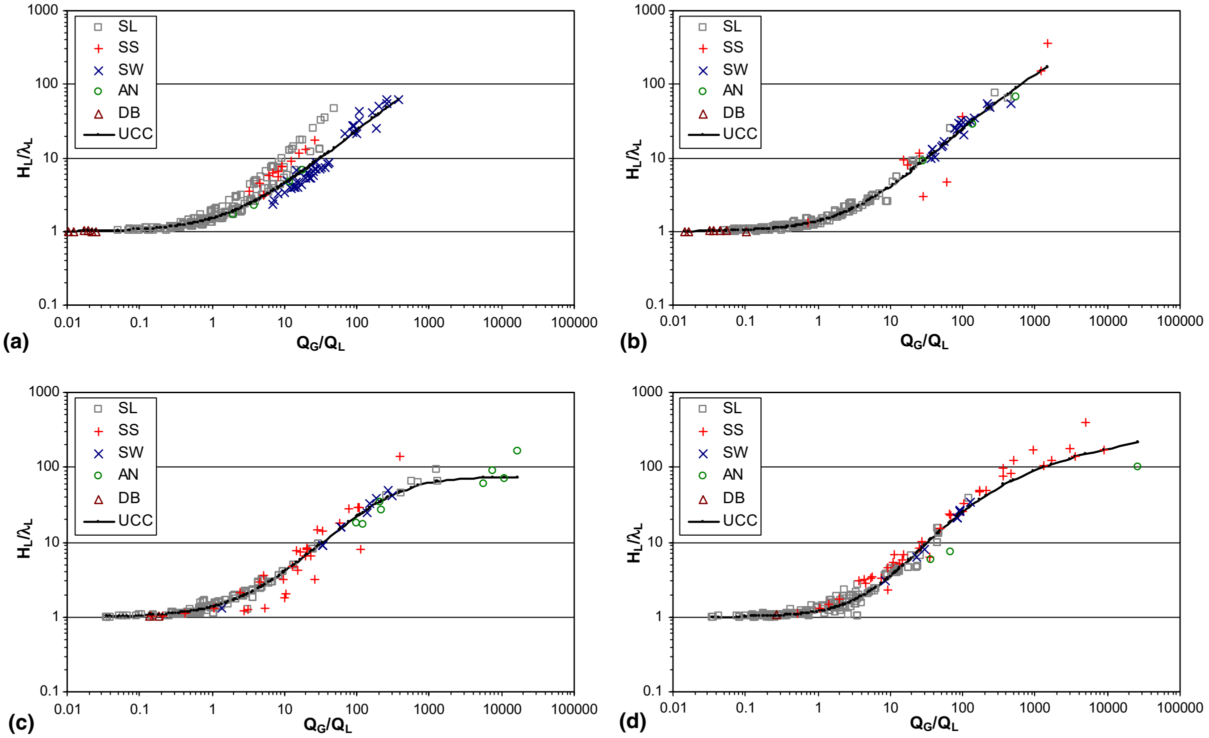

Fig. 4. Universal composite correlations for (a) $Re < 2000$, (b) $2000 \leqslant Re < 5000$, (c) $5000 \leqslant Re < 10\,000$, (d) $10\,000 \leqslant Re < 20\,000$.

by ranges of mixture Reynolds number and composite correlations were created for each flow type. The parameters a, b, c, d and t of each correlation are presented in Table 9.

The statistical parameters E_1–E_8 for each correlation are presented in Table 10.

The slug flow holdup correlations have an average error of −3.5% and an average absolute error of 12.7%. 79.5% of the points (1038 experimental points) are in the band between ±20%. The dispersed bubble flow holdup correlation has an average error of −0.2 and an average absolute error of 0.7%. 85% of the points (17 points) are in the band between ±1.4%. The average error for stratified flow is −1.1% and the average absolute error is 29.4%. 82.7% of the 692 points (572 points) are in the band between ±42.3%. The average error for annular flow is −4.9% and the average absolute error of 28.1%. However, only 75.3% of the 259 points (195 points) are in the band between ±39.8%.

In slug flow a better fit is obtained for $Re \leqslant 139$. However, the application range is very limited ($0.33 \leqslant Q_G/Q_L \leqslant 48.8$). The parameters a, b, c, d and t for this correlation are presented in Table 11.

The statistical parameters E_1–E_8 for slug flow holdup correlation, $Re \leqslant 139$ and $0.33 \leqslant Q_G/Q_L \leqslant 48.8$ are presented in Table 12.

The experiments for $Re \leqslant 139$ are just a few (50 experimental points) and the zone for $Q_G/Q_L > 48.8$ is not defined.

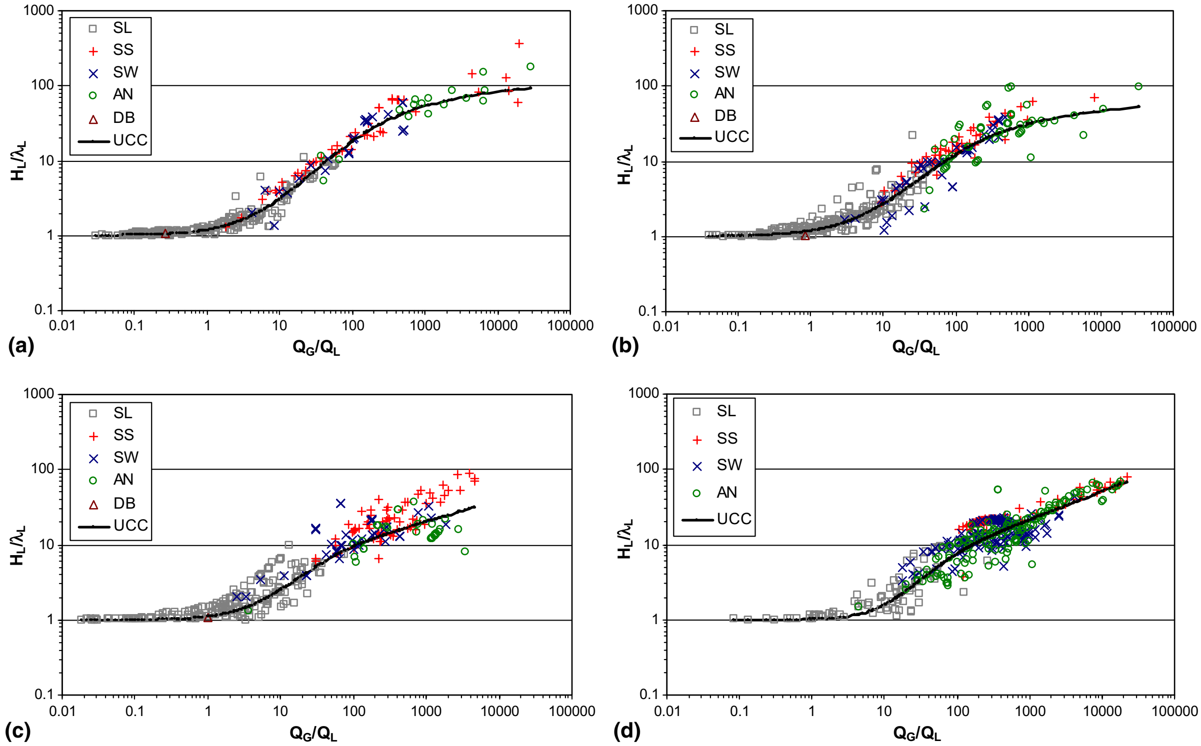

Fig. 5. Universal composite correlations for (a) $20\,000 \leqslant Re < 40\,000$, (b) $40\,000 \leqslant Re < 100\,000$, (c) $100\,000 \leqslant Re < 300\,000$, (d) $Re \geqslant 300\,000$.

Table 8
Statistical parameters of universal composite correlations

Range	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
$Re < 2000$	−6.55	18.11	25.07	25.92	−1.16	1.75	4.11	4.27
$2000 \leqslant Re < 5000$	−0.01	10.75	25.95	25.95	−1.16	2.10	13.89	13.94
$5000 \leqslant Re < 10\,000$	2.47	14.35	29.12	29.22	−1.49	2.45	10.77	10.88
$10\,000 \leqslant Re < 20\,000$	−2.47	13.28	22.77	22.91	−2.22	3.72	20.68	20.80
$20\,000 \leqslant Re < 40\,000$	−2.37	15.26	22.91	23.04	−2.66	4.10	18.45	18.64
$40\,000 \leqslant Re < 100\,000$	−3.92	21.39	33.34	33.57	−1.93	2.92	7.70	7.94
$100\,000 \leqslant Re < 300\,000$	−10.25	26.26	35.57	37.02	−2.85	4.01	8.69	9.14
$300\,000 \leqslant Re < 2\,670\,000$	−4.58	32.74	44.98	45.21	−2.40	4.62	5.92	6.39

5. Performance comparison of holdup correlations and models from various sources against the 2276 experimental data

In this section we evaluate the performance of the 2276 holdup experimental points of our correlations and the correlations and the models in the literature. The models and the correlations evaluated are indexed as follows:

Table 9
Parameters of the holdup correlations for each flow pattern

FP	Range	a	b	c	d	t
SL	$Re < 2000$	85.8986	0.2236	0.8079	0.2549	115.1514
	$2000 \leqslant Re < 10\,000$	87.4521	0.1197	0.8812	0.2300	100.3408
	$10\,000 \leqslant Re < 100\,000$	29.9532	0.8411	0.1036	0.01483	103.4254
	$100\,000 \leqslant Re < 300\,000$	22.2924	0.5506	0.3033	0.04402	103.0586
	$300\,000 \leqslant Re < 1\,600\,000$	16.4879	0.7116	0.03704	0.01333	103.2149
DB	$Re \leqslant 40\,000$	0.4189	0.7474	15.3580	0.6820	−0.6416
ST	$Re < 40\,000$	72.6460	0.07633	1.0797	0.3618	100.2523
	$40\,000 \leqslant Re < 100\,000$	10.9333	0.2091	99.9363	0.8191	0.9032
	$100\,000 \leqslant Re < 300\,000$	7.6656	0.3091	0.3142	0.5750	166.0573
	$300\,000 \leqslant Re < 1\,970\,000$	5.5983	0.3424	0.02663	0.6472	100.0544
AN	$Re < 40\,000$	29.3073	0.1273	0.7551	0.7236	194.0617
	$40\,000 \leqslant Re < 100\,000$	5.1204	0.2441	1.9999	3.6295	150.4515
	$100\,000 \leqslant Re < 300\,000$	11.5159	0.02339	2.6863	2.0494	153.9041
	$300\,000 \leqslant Re < 2\,670\,000$	4.4926	0.3743	0.3804	0.3223	155.4373

Table 10
Statistical parameters of the holdup correlations for each flow pattern

FP	Range	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
SL	$Re < 2000$	−7.61	12.20	17.70	19.28	−0.89	1.03	3.18	3.30
	$2000 \leqslant Re < 10\,000$	−1.32	7.34	11.27	11.35	−0.19	0.55	2.73	2.74
	$10\,000 \leqslant Re < 100\,000$	−2.31	11.60	18.16	18.30	−0.23	0.40	1.35	1.37
	$100\,000 \leqslant Re < 300\,000$	−5.41	18.19	25.85	26.41	−0.36	0.55	1.03	1.09
	$300\,000 \leqslant Re < 1\,600\,000$	−6.46	27.46	37.92	38.48	−0.80	1.18	1.92	2.09
DB	$Re \leqslant 40\,000$	−0.24	0.75	1.36	1.39	0.00	0.01	0.01	0.01
ST	$Re < 40\,000$	−1.67	29.06	43.40	43.43	−6.02	10.22	32.77	33.32
	$40\,000 \leqslant Re < 100\,000$	5.75	29.28	48.64	48.98	−1.37	3.59	5.15	5.33
	$100\,000 \leqslant Re < 300\,000$	−8.49	28.49	35.50	36.51	−4.39	6.94	9.73	10.69
	$300\,000 \leqslant Re < 1\,970\,000$	0.26	30.09	41.18	41.18	−1.78	4.88	5.57	5.85
AN	$Re < 40\,000$	−6.05	23.93	29.58	30.21	−9.48	13.64	22.76	24.71
	$40\,000 \leqslant Re < 100\,000$	−8.35	30.91	43.88	44.69	−7.09	9.94	16.67	18.14
	$100\,000 \leqslant Re < 300\,000$	−13.62	23.78	27.72	30.98	−3.37	4.28	5.95	6.87
	$300\,000 \leqslant Re < 2\,670\,000$	−1.32	28.98	41.92	41.94	−1.18	4.82	8.19	8.28

Table 11
Parameters of the holdup correlations for slug flow, $Re \leqslant 139$ and $0.33 \leqslant Q_G/Q_L \leqslant 48.8$

Range	a	b	c	d	t
$Re \leqslant 139$	85.8992	0.3717	0.7275	0.3116	115.1512

Table 12
Statistical parameters of the holdup correlations for slug flow, $Re \leqslant 139$ and $0.33 \leqslant Q_G/Q_L \leqslant 48.8$

FP	Range	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
SL	$Re \leqslant 139$	5.64	9.04	16.71	17.66	0.15	0.31	0.52	0.54

HOM (homogeneous model),
ARM (Armand, 1946),
L&M (Lockhart and Martinelli, 1949),
FLA (Flanigan, 1958),
HOO (Hoogendoorn, 1959),
LEV (Levy, 1960),
HUG (Hughmark, 1962),
BAR (Transformed Baroczy correlation, Butterworth, 1975),
N&Y (Nishino and Yamazaki, 1963),
THO (Transformed Thom correlation, Butterworth, 1975),
ZIV (Zivi, 1963),
T&W (Turner and Wallis, 1965),
GUZ (Guzhov et al., 1967),
EAT (Eaton et al., 1967),
BON (Bonnecaze et al., 1971),
B&B (Beggs and Brill, 1973),
M&G (Mattar and Gregory, 1974),
GRE (Gregory et al., 1978),
C&SC (Chen and Spedding, 1981),
C&SM (Chen and Spedding model, Chen and Spedding, 1983; Spedding and Chen, 1984),
TAN (Tandon et al., 1985),
M&B (Minami and Brill, 1987),
HAR (Hart et al., 1989),
ABD (Abdul-Majeed, 1996),
SPE (Spedding et al., 1998),
GOM (Gómez et al., 2000),
TMC (theoretical model correlations, Eq. (3), Table 5),
UCHC (universal composite holdup correlations, Eq. (13), Table 7),
FPHC (composite holdup correlations sorted by flow pattern, Eq. (13), Table 9).

The comparison of the accuracy of holdup prediction of the correlations and the models from different authors against 2276 points is shown in Table 13, that also includes the statistical parameters E_1–E_8 for each correlation.

The performance of our correlation FPHC sorted by flow pattern is the best with an average absolute error of 19.0%. The universal correlations UCHC in which the flow pattern are ignored is second best and the theoretical models TMC is the third best with average absolute errors of 21.0% and 24.0%, respectively. The Beggs and Brill (1973) correlation obtain the fourth best performance with average absolute error of 31.9%. The homogeneous model and the models and the

Table 13
The comparison of the accuracy of holdup prediction of the correlations and the models from different authors against 2276 points

Model or correlation	Statistical parameters							
	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
FPHC	−2.1	19.0	30.7	30.7	−0.02	0.05	0.08	0.08
UCHC	−4.1	21.0	33.0	33.2	−0.03	0.05	0.09	0.10
TMC	−5.5	24.0	35.2	35.6	−0.03	0.06	0.10	0.10
BBC	−26.5	31.9	31.3	41.0	−0.08	0.09	0.11	0.14
N&Y	7.7	31.9	51.4	52.0	−0.01	0.07	0.12	0.12
BAR	9.5	33.5	47.6	48.6	0.04	0.09	0.12	0.12
C&SM	15.2	36.0	64.1	65.9	−0.02	0.09	0.14	0.15
ABD	−8.4	37.8	68.1	68.7	−0.08	0.16	0.22	0.24
L&M	30.3	40.5	95.5	100.2	0.02	0.07	0.09	0.10
THO	−11.0	44.9	54.3	55.4	0.03	0.11	0.14	0.15
ZIV	2.4	45.3	57.7	57.7	0.06	0.13	0.16	0.17
LEV	−40.4	46.0	40.4	57.1	−0.07	0.10	0.12	0.14
TAN	28.2	53.2	70.1	75.5	0.03	0.21	0.31	0.31
EAT	52.1	55.5	65.5	83.8	0.11	0.12	0.12	0.16
HOM	−57.1	57.1	35.8	67.4	−0.14	0.14	0.14	0.20
HUG	44.6	59.9	137.2	144.3	−0.01	0.06	0.09	0.09
SPE	57.0	68.5	88.9	105.6	0.18	0.20	0.22	0.29
M&B	82.9	84.5	104.7	133.5	0.12	0.13	0.11	0.16
HOO	126.9	134.3	325.2	349.1	0.07	0.11	0.14	0.16
T&W	155.0	155.3	266.9	308.7	0.21	0.22	0.15	0.26
C&SC	157.7	159.1	251.1	296.6	0.20	0.20	0.14	0.24
BON	145.7	167.3	524.5	544.4	−0.02	0.10	0.14	0.14
ARM	145.7	167.3	524.5	544.4	−0.02	0.10	0.14	0.14
GUZ	178.0	193.1	599.3	625.2	0.01	0.10	0.13	0.14
GRE	306.2	312.9	690.4	755.3	0.21	0.27	0.25	0.33
GOM	457.4	462.7	1740.8	1799.9	0.34	0.36	0.29	0.44
HAR	634.7	634.8	1103.7	1273.2	0.51	0.51	0.29	0.59
M&G	790.9	844.0	2426.9	2552.5	0.07	0.50	0.56	0.56
FLA	1111.8	1127.1	3205.3	3392.7	0.40	0.53	0.48	0.62

correlations developed by Armand (1946), Flanigan (1958), Hoogendoorn (1959), Hughmark (1962), Turner and Wallis (1965), Guzhov et al. (1967), Eaton et al. (1967), Bonnecaze et al. (1971), Mattar and Gregory (1974), Gregory et al. (1978), Chen and Spedding (1981), Tandon et al. (1985), Minami and Brill (1987), Spedding et al. (1998), Hart et al. (1989) and Gómez et al. (2000) obtain average absolute errors higher to 50%.

We turn now to an evaluation of the predictors when the experimental data is sorted by flow pattern. The following data were used: 1305 slug flow data points, 20 dispersed bubble data points, 692 stratified flow data points and 259 annular flow data points. The statistical parameters E_1–E_8 for each flow pattern are presented in Tables 14–17. This kind of comparison naturally favors correlations like FPHC, which recognize the flow pattern.

The correlations FPHC developed in this work which have been sorted by flow type, again show the best performance with average absolute errors of 12.0%, 0.7%, 29.4% and 28.1% for slug

Table 14
Evaluation of the correlations and the models using slug flow data

Model or correlation	Statistical parameters							
	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
FPHC	−2.2	12.0	19.5	19.7	−0.02	0.05	0.09	0.09
UCHC	−3.5	13.3	21.9	22.2	−0.04	0.06	0.11	0.12
HUG	−2.1	14.4	24.2	24.3	−0.04	0.06	0.10	0.10
TMC	2.3	14.7	24.5	24.6	−0.01	0.06	0.09	0.09
BBC	−15.6	20.3	23.4	28.2	−0.09	0.10	0.12	0.16
L&M	15.1	20.5	31.6	35.0	0.04	0.08	0.10	0.11
GUZ	0.9	20.8	49.7	49.7	−0.05	0.08	0.12	0.13
C&SM	−4.6	20.9	44.3	44.6	−0.07	0.09	0.13	0.15
BON	−4.6	20.9	44.3	44.6	−0.07	0.09	0.13	0.15
ARM	−4.6	20.9	44.3	44.6	−0.07	0.09	0.13	0.15
HOO	11.3	21.0	39.9	41.5	0.02	0.08	0.12	0.12
N&Y	8.2	21.3	36.0	36.9	0.01	0.09	0.13	0.13
LEV	−19.0	25.3	29.3	34.9	−0.07	0.11	0.13	0.15
BAR	24.1	29.6	40.5	47.2	0.08	0.11	0.12	0.15
THO	22.3	31.3	37.3	43.5	0.10	0.14	0.14	0.17
HOM	−34.4	34.4	28.0	44.4	−0.16	0.16	0.15	0.22
ABD	−20.7	34.7	36.0	41.5	−0.15	0.22	0.23	0.27
ZIV	33.0	39.2	44.8	55.6	0.14	0.17	0.15	0.21
EAT	37.7	39.5	40.6	55.4	0.15	0.16	0.11	0.18
M&B	40.8	42.5	49.3	64.0	0.14	0.16	0.11	0.18
TAN	24.1	54.0	62.0	66.5	0.02	0.30	0.38	0.38
GRE	44.1	54.4	91.4	101.5	0.11	0.20	0.22	0.25
C&SC	68.1	68.6	85.8	109.6	0.22	0.22	0.15	0.26
T&W	72.4	72.7	84.5	111.3	0.24	0.25	0.15	0.29
SPE	76.0	78.6	90.4	118.1	0.28	0.30	0.22	0.36
GOM	82.0	86.6	129.1	153.0	0.26	0.29	0.25	0.36
M&G	22.7	108.6	220.4	221.5	−0.24	0.46	0.47	0.53
FLA	85.4	111.3	261.7	275.3	0.10	0.32	0.38	0.39
HAR	139.7	139.7	261.0	296.0	0.39	0.39	0.26	0.47

flow, dispersed bubble flow, stratified flow and annular flow, respectively. The universal correlations UCHC present the second best performance for slug flow and stratified flow with average absolute errors of 13.3% and 31.3%, respectively. For annular flow the Abdul-Majeed (1996) correlation show the second best performance with an average absolute error of 32.3%, followed in third place by the universal correlations UCHC with an average absolute error of 34.9%. For dispersed bubble flow the Beggs and Brill (1973) correlation obtain the second best performance with an average absolute error of 1.4%. Although for dispersed bubble flow the universal correlations UCHC fall to ninth place, these have an average absolute error of 1.9%.

6. Summary and conclusions

Data from 2276 gas–liquid flow experiments in horizontal pipelines were sorted by flow pattern and a data structure suitable for the study of liquid holdup was created.

Table 15
Evaluation of the correlations and the models using dispersed bubble flow data

Model or correlation	Statistical parameters							
	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
FPHC	−0.2	0.7	1.4	1.4	0.00	0.01	0.01	0.01
BBC	−0.9	1.4	2.1	2.3	−0.01	0.01	0.02	0.02
C&SM	0.5	1.7	3.3	3.4	0.00	0.01	0.02	0.02
BON	0.5	1.7	3.3	3.4	0.00	0.01	0.02	0.02
ARM	0.5	1.7	3.3	3.4	0.00	0.01	0.02	0.02
HOM	−1.8	1.8	2.1	2.8	−0.01	0.01	0.02	0.02
GUZ	0.9	1.9	3.9	4.0	0.00	0.01	0.02	0.02
HUG	1.3	1.9	3.5	3.7	0.01	0.01	0.02	0.02
UCHC	1.3	1.9	2.9	3.2	0.01	0.02	0.02	0.02
TMC	−0.2	2.2	4.0	4.0	−0.01	0.02	0.03	0.03
HOO	2.6	3.0	5.6	6.2	0.02	0.02	0.03	0.04
N&Y	4.2	4.4	9.5	10.4	0.03	0.03	0.05	0.06
LEV	−4.4	4.7	9.5	10.5	−0.03	0.03	0.05	0.06
BAR	4.6	4.9	10.9	11.9	0.03	0.03	0.06	0.07
L&M	−2.7	5.4	5.9	6.6	−0.03	0.05	0.04	0.05
M&B	6.8	6.8	11.5	13.4	0.05	0.05	0.06	0.08
GOM	7.2	7.2	12.8	14.8	0.05	0.05	0.07	0.09
THO	7.3	7.3	13.8	15.7	0.05	0.05	0.08	0.09
EAT	7.4	7.4	12.6	14.7	0.05	0.05	0.07	0.09
C&SC	6.1	7.6	16.6	17.8	0.04	0.05	0.09	0.10
T&W	8.2	8.4	18.5	20.3	0.05	0.05	0.10	0.12
ZIV	8.5	8.5	16.7	18.8	0.06	0.06	0.09	0.11
ABD	−8.0	9.7	16.0	18.0	−0.06	0.07	0.11	0.13
GRE	−2.5	9.9	13.2	13.4	−0.04	0.08	0.10	0.11
HAR	11.9	11.9	24.8	27.6	0.08	0.08	0.14	0.16
SPE	11.9	11.9	24.8	27.6	0.08	0.08	0.14	0.16
FLA	−27.0	27.5	15.5	31.8	−0.26	0.27	0.16	0.31
TAN	−74.3	82.0	47.5	89.8	−0.74	0.79	0.43	0.88
M&G	−91.2	91.2	18.1	95.3	−0.86	0.86	0.23	0.92

This paper proposes a theoretical model which relates the liquid holdup and no-slip liquid holdup (H_L/λ_L) with the gas–liquid volumetric flow rates relation (Q_G/Q_L) for different mixture Reynolds numbers (Re) ranges. This model is based on the developed works by Butterworth (1975) and Chen and Spedding (1983). The Reynolds number appropriate for gas–liquid flows in horizontal pipes is based on the mixture velocity and the liquid kinematic viscosity. The proposed theoretical model fits the experimental data well for values of $Q_G/Q_L < 10$ when $Re < 300\,000$. However, for $Q_G/Q_L < 1$ when $Re \geqslant 300\,000$ and $Q_G/Q_L > 10$ for all Re, the theoretical model does not perform well.

The experimental values of H_L/λ_L vs. Q_G/Q_L for different Reynolds numbers ranges are described by composite power laws generated by logistic dose curves. Three regions are clearly defined by distributions of the experimental values.

The first region includes experiments where $Q_G/Q_L \ll 1$. In this region, H_L/λ_L approaches 1 for all Re.

Table 16
Evaluation of the correlations and the models using stratified flow data

Model or correlation	Statistical parameters							
	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
FPHC	−1.1	29.4	42.3	42.3	−0.02	0.05	0.08	0.08
UCHC	−12.0	31.3	38.4	40.3	−0.03	0.05	0.08	0.08
TMC	−22.1	36.4	38.4	44.3	−0.04	0.06	0.08	0.09
BAR	−19.2	37.2	40.1	44.5	−0.03	0.06	0.09	0.10
N&Y	−8.5	39.4	50.6	51.3	−0.05	0.06	0.09	0.10
TAN	25.6	44.2	71.7	76.1	0.05	0.09	0.16	0.17
L&M	23.1	44.8	81.3	84.5	0.00	0.06	0.09	0.09
ABD	16.8	46.7	105.9	107.2	0.04	0.09	0.20	0.20
SPE	21.6	48.0	70.5	73.7	0.04	0.09	0.15	0.15
BBC	−43.2	49.6	34.4	55.2	−0.08	0.08	0.09	0.12
ZIV	−40.9	54.1	44.8	60.6	−0.05	0.08	0.10	0.12
C&SM	49.8	59.9	77.1	91.8	0.08	0.09	0.14	0.16
THO	−57.2	64.6	38.5	69.0	−0.07	0.09	0.10	0.12
EAT	70.0	75.6	81.2	107.3	0.08	0.09	0.12	0.14
LEV	−69.7	75.7	36.5	78.8	−0.09	0.11	0.10	0.14
HOM	−89.0	89.0	14.2	90.2	−0.14	0.14	0.12	0.19
HUG	81.3	97.2	151.2	171.7	0.01	0.07	0.09	0.09
M&B	128.2	129.6	112.7	170.8	0.12	0.12	0.10	0.16
T&W	191.9	192.1	208.3	283.3	0.18	0.18	0.14	0.23
BON	216.9	238.6	546.1	587.6	0.02	0.10	0.13	0.13
ARM	216.9	238.6	546.1	587.6	0.02	0.10	0.13	0.13
C&SC	248.7	249.0	297.1	387.6	0.20	0.20	0.12	0.23
HOO	273.3	276.1	429.5	509.2	0.17	0.17	0.15	0.23
GUZ	270.0	281.3	621.8	678.0	0.07	0.11	0.12	0.14
GRE	671.0	671.0	939.2	1154.5	0.44	0.44	0.19	0.48
GOM	673.4	674.1	1529.1	1671.0	0.47	0.47	0.26	0.54
HAR	1016.3	1016.4	1071.2	1477.2	0.69	0.69	0.21	0.72
M&G	1218.2	1225.4	2470.6	2755.0	0.46	0.51	0.29	0.55
FLA	1728.5	1728.5	3288.4	3715.6	0.80	0.80	0.18	0.82

The second region includes data where $Q_G/Q_L > 1$ and approximately $Q_G/Q_L < 1000$ for $Re < 300\,000$. This is a transition region in which the values of H_L/λ_L increase significantly with the increment of Q_G/Q_L. In this region a marked effect of slip between the phases exists. For $Re > 300\,000$, the superior limit of the transition region is $Q_G/Q_L \gg 1000$, but this limit could not be established accurately because we did not have enough experimental data.

The third region includes experiments where $Q_G/Q_L > 1000$ for $Re < 300\,000$. In this region of high gas flow rate and low liquid flow rate, the dominant flow patterns are stratified wavy and annular flow. In this region a marked increment of H_L/λ_L with the increment of Q_G/Q_L is not evidenced. Spedding and Chen (1984) affirm that for $Q_G/Q_L > 15\,000$, H_L remains constant. This would imply that H_L/λ_L should diminish with the increment of Q_G/Q_L since λ_L increases. However, this tendency is not observed in this work. This could be due to the lack of experimental data in this region when $Re < 300\,000$. However, although there are several experiments for

Table 17
Evaluation of the correlations and the models using annular flow data

Model or correlation	Statistical parameters							
	E_1 [%]	E_2 [%]	E_3 [%]	E_4 [%]	E_5	E_6	E_7	E_8
FPHC	−4.9	28.1	39.5	39.8	−0.01	0.02	0.03	0.03
ABD	−13.7	32.3	42.9	45.0	−0.02	0.02	0.05	0.05
UCHC	13.8	34.9	52.1	53.8	0.00	0.02	0.03	0.03
TMC	6.1	35.0	50.9	51.3	−0.01	0.02	0.04	0.04
BAR	13.2	45.1	63.8	65.2	0.00	0.03	0.05	0.05
BBC	−38.9	45.4	33.7	51.6	−0.03	0.03	0.05	0.06
C&SM	24.2	50.6	71.2	75.2	0.00	0.03	0.05	0.05
ZIV	−36.8	55.2	50.5	62.5	−0.02	0.03	0.06	0.06
THO	−56.5	63.2	38.5	68.5	−0.03	0.04	0.05	0.06
N&Y	48.3	67.3	86.7	99.3	0.00	0.03	0.05	0.05
TAN	63.2	71.0	88.3	108.6	0.03	0.04	0.07	0.08
LEV	−72.6	73.9	26.6	77.4	−0.04	0.04	0.04	0.06
SPE	59.5	77.3	98.3	114.9	0.03	0.05	0.10	0.10
EAT	80.4	86.2	94.9	124.5	0.03	0.04	0.05	0.06
HOM	−90.5	90.5	10.6	91.3	−0.06	0.06	0.06	0.08
L&M	128.7	132.6	215.8	251.3	0.03	0.04	0.04	0.05
M&B	179.4	181.9	161.4	241.6	0.06	0.07	0.06	0.09
HUG	185.5	193.6	258.9	318.7	0.03	0.04	0.04	0.05
HOO	327.5	336.7	513.8	609.6	0.05	0.07	0.06	0.08
C&SC	378.1	387.3	411.0	559.0	0.10	0.12	0.11	0.15
T&W	484.4	484.4	572.0	750.2	0.16	0.16	0.11	0.19
GRE	676.4	682.0	974.3	1186.8	0.15	0.16	0.13	0.20
BON	723.9	726.8	1073.4	1295.5	0.11	0.12	0.06	0.13
ARM	723.9	726.8	1073.4	1295.5	0.11	0.12	0.06	0.13
GUZ	837.9	839.9	1224.1	1484.3	0.13	0.14	0.06	0.15
GOM	1806.8	1828.6	4206.4	4579.4	0.40	0.41	0.35	0.53
HAR	2157.6	2157.6	1826.1	2829.8	0.67	0.67	0.26	0.72
M&G	3588.4	3588.6	4893.7	6072.5	0.68	0.68	0.10	0.69
FLA	4723.6	4723.6	6460.6	8008.6	0.89	0.89	0.13	0.90

$300\,000 \leqslant Re < 2\,670\,000$, this tendency is not observed. It is possible that for high mixture Reynolds numbers the third region may exist for values of $Q_G/Q_L \gg 15\,000$.

Two thousand two hundred and seventy-six experimental values were fit to composite power law correlations, H_L/λ_L vs. Q_G/Q_L in which power laws are joined by logistic dose curves. The correlations that ignore flow types are called universal. Composite correlations which depend on the flow type were also generated The Reynolds number range for each correlation was selected to minimize the spread of experimental data.

Power law correlations were determined for subsets of the 2276 points corresponding to stratified, slug, disperse bubble and annular flow. Composite power laws are very practical because the transition region is predicted to a statistical accuracy consistent with spread of the data.

The predictions of the correlations developed in this work were tested for the spread of the actual data against the predictions. The same tests were carried out for the correlations sorted by flow type. The standard deviations are small for bubble flow (1.4%) and slug flow (19.5%).

The prediction of our correlations were also tested against correlations and models from the literature. The composite correlations sorted by flow type are more accurate than any other predictor for all cases ($E_2 = 19\%$). The universal composite correlation is second best in the data set in which all flow types are included ($E_2 = 21.0\%$), followed in third place by the theoretical model ($E_2 = 24\%$). Although, the Beggs and Brill (1973) correlation present the fourth best performance ($E_2 = 31.9\%$), for annular flow and stratified flow falls to sixth ($E_2 = 45.4\%$) and tenth ($E_2 = 49.6\%$) place, respectively. In general, the Flanigan (1958) correlation has the biggest errors.

Although the correlations developed in this work show the best performance in stratified flow and annular flow, the average absolute errors for the FPHC correlations are 29.4% and 28.1%, respectively. In these flow types the effect of the gravity and superficial tension, neglected in this paper, could be important. Including this effect through of Froude number, Morton number or Weber number, could give rise to improved holdup correlations.

Universal (independent of flow type) and composite (for all Reynolds numbers) correlations are very useful for field operations for which the flow type may not be known. It is a best guess for the liquid holdup when the flow type is unknown or different flow types are encountered in one line.

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Appendix A. Previous holdup correlations and models

This appendix summarizes 26 literature holdup models and correlations including the homogeneous model that are compared in this work with the proposed model.

The holdup H_L in the homogeneous model for two-phase flow could be expressed as

$$H_L = \lambda_L = \frac{Q_L}{Q_L + Q_G} \tag{A.1}$$

where λ_L is the no-slip holdup, Q_L and Q_G are the gas and liquid flow rates, respectively.

Armand (1946) derived a simple holdup correlation for two-phase flow in horizontal pipes given by

$$\frac{H_G}{H_L} = \frac{1}{0.2 + 1.2(Q_G/Q_L)} \tag{A.2}$$

which is a special case of the theoretical development due to Nguyen and Spedding (1977) and is recommended for bubble and slug flows (Spedding and Chen, 1984). $H_G = 1 - H_L$ is the void fraction.

Lockhart and Martinelli (1949) holdup correlation has the following form:

$$\frac{H_L}{H_G} = f(X) \tag{A.3}$$

where $X = [(dp/dl)_{SL}/(dp/dl)_{SG}]^{1/2}$ is the Lockhart and Martinelli parameter.

Butterworth (1975) showed that the Holdup Lockhart and Martinelli correlation is closely approximated by

$$\frac{H_L}{H_G} = 0.28X^{0.71} \tag{A.4}$$

Flanigan (1958) developed a holdup correlation based in field data acquired on a pipe with an inner diameter of 16 in. The liquid holdup correlation is given by

$$H_L = \frac{1}{1 + 0.3264U_{SL}^{1.006}} \tag{A.5}$$

where U_{SL} is the superficial liquid velocity.

Hoogendoorn (1959) derived an implicit equation to evaluate the void fraction in horizontal pipes,

$$\frac{H_G}{H_L} = 0.60\left[U_{SG}\left(1 - \frac{H_G}{1 - H_G}\frac{U_{SL}}{U_{SG}}\right)\right]^{0.85} \tag{A.6}$$

where U_{SL} and U_{SG} are the superficial velocities of the liquid and gas phases in meter by second, respectively.

Levy (1960) developed a simple correlation from theoretical considerations,

$$H_G = \frac{\phi_L - 1}{\phi_L} \tag{A.7}$$

where $\phi_L = [(dp/dl)_{TP}/(dp/dl)_{SL}]^{1/2}$ is the Lockhart and Martinelli parameter, which is evaluated with the Chisholm (1967) correlation.

Hughmark (1962) presented a void fraction correlation based on the Bankoff (1960) correlation,

$$\frac{1}{x} = 1 - \frac{\rho_L}{\rho_G}\left(1 - \frac{K}{H_G}\right) \tag{A.8}$$

or

$$K = \frac{H_G}{\lambda_G} \tag{A.9}$$

where x is the quality, ρ_L and ρ_G are the liquid and gas density, respectively, $\lambda_G = Q_G/(Q_L + Q_G)$ is the no-slip void fraction and K is a dimensionless flow parameter.

Hughmark (1962) used several sources of data to correlate K and found that it could be correlated against a variable $Z = Re^{1/6}Fr^{1/8}\lambda_L^{-1/4}$, where $Re = GD/(H_L\rho_L + H_G\rho_G)$ is the Reynolds number and $Fr = (U_{SL} + U_{SG})^2/(gD)$ is the Froude number. G is the total mass flux per unit area and g is the gravitational acceleration. Hughmark (1962) correlation requires an iterative procedure to obtain H_G. Although this correlation was also developed for vertical flow, it is widely used for horizontal flow applications (Brill and Beggs, 1988). We found that the dimensionless flow parameter K could be adjusted by a fifth order logarithmic equation.

$$K = 0.1746 - 0.1301\ln(Z) + 0.7508\ln(Z)^2 - 0.4308\ln(Z)^3 + 0.09553\ln(Z)^4 - 0.007452\ln(Z)^5 \quad (A.10)$$

Butterworth (1975) developed an equation that has an excellent agreement with Baroczy's curves (1963) for void fractions less than 0.9.

$$\frac{H_L}{H_G} = \left(\frac{1-x}{x}\right)^{0.74}\left(\frac{\rho_G}{\rho_L}\right)^{0.65}\left(\frac{\mu_L}{\mu_G}\right)^{0.13} \quad (A.11)$$

Nishino and Yamazaki (1963) presented a simple void fraction model,

$$H_G = 1 - \left[\frac{(1-x)\rho_G}{x\rho_L + (1+x)\rho_G}\right]^{1/2} \quad (A.12)$$

Zivi (1963) derived simple void fraction models of which the simplest is defined as

$$\frac{H_L}{H_G} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{2/3} \quad (A.13)$$

Butterworth (1975) showed that the Thom (1964) correlation may be approximated by

$$\frac{H_L}{H_G} = \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.89}\left(\frac{\mu_L}{\mu_G}\right)^{0.18} \quad (A.14)$$

Turner and Wallis (1965) developed a void fraction models based in the Lockhart and Martinelli parameter X, which for turbulent flow may be written as (Butterworth, 1975)

$$\frac{H_L}{H_G} = X^{0.8} \quad (A.15)$$

Guzhov et al. (1967) derived a void fraction correlation for transportation in gas–liquid systems.

$$\frac{H_G}{\lambda_G} = 0.81[1 - \exp(-2.2\sqrt{Fr})] \quad (A.16)$$

Eaton et al. (1967) developed a correlation to evaluate the holdup in horizontal pipes. The holdup was correlated with the following dimensionless group $\frac{1.84N_{USL}^{0.575}}{N_{USG}N_D^{0.0277}}\left[\frac{p}{p_{atm}}\right]^{0.05}N_L^{0.1}$, based in the liquid velocity number N_{USG}, the gas velocity number N_{USG}, the pipe diameter number N_D and the liquid viscosity number N_L defined by Ros (1961), where p is the system pressure and p_{atm} is the reference atmospheric pressure (101 008 Pa). We found that the Eaton et al. (1967) correlation could be adjusted by

$$H_L = \frac{Z}{0.2578 + 0.9555Z + 0.1397Z^{1/2}} \quad (A.17)$$

Bonnecaze et al. (1971) derived a correlation to evaluate the holdup for two-phase slug flow in inclined pipes.

$$H_L = 1 - \frac{(1-\lambda_L)}{1.2 + 0.35(1-\rho_G/\rho_L)\delta/\sqrt{Fr}} \tag{A.18}$$

where $\delta = 0$ for horizontal flow, $\delta = 1$ for uphill flow and $\delta = -1$ for downhill flow.

Beggs and Brill (1973) formulated a correlation to evaluate the holdup for two-phase flow in inclined pipes.

$$H_L = H_{L(0)}\psi \tag{A.19}$$

where $H_{L(0)}$ is the holdup which would exist at the same conditions in a horizontal pipe and ψ is the correction factor for the effect of pipe inclination. $\psi = 1$ for horizontal pipes. $H_{L(0)}$ is given by

$$H_{L(0)} = \frac{a\lambda_L^b}{Fr^c} \tag{A.20}$$

where a, b and c are flow pattern dependent parameters.

Mattar and Gregory (1974) generated a correlation to evaluate the holdup for air–oil slug flow in an upward–inclined pipe.

$$H_L = 1 - \frac{U_{SG}}{1.3(U_{SG} + U_{SL}) + 0.7} \tag{A.21}$$

Gregory et al. (1978) developed a simple correlation to evaluate the holdup in the slug for horizontal gas–liquid slug flow.

$$H_{LLS} = \frac{1}{1 + \left(\frac{U_M}{8.66}\right)^{1.39}} \tag{A.22}$$

where the mixture velocity U_M has units of meters per second.

Chen and Spedding (1981) developed a correlation to determine the holdup for stratified and annular flow based in the Lockhart–Martinelli parameter. Chen and Spedding (1981) recommend to use the following correlation for stratified flow:

$$H_L = \frac{X^{2/3}}{1 + X^{2/3}} \tag{A.23}$$

Chen and Spedding (1981) propose to include an experimental adjustment factor k_i to improve the performance of the correlation in annular flow,

$$H_L = \frac{X^{2/3}}{k_i + X^{2/3}} \tag{A.24}$$

where $k_i = 2.5$ for big diameter pipes ($D \geqslant 0.2$ m), $k_i = 6$ for small diameter pipes ($D \leqslant 0.045$ m), while $k_i = 1$ for diameter pipes between 0.045 m and 0.2 m.

Chen and Spedding (1983) carried out a theoretical study of the correlation proposal by Butterworth (1975),

$$\frac{H_L}{H_G} = A\left[\frac{1-x}{x}\right]^p \left[\frac{\rho_G}{\rho_L}\right]^q \left[\frac{\mu_L}{\mu_G}\right]^r \tag{A.25}$$

where A, p, q and r are dependent parameters of flow pattern considered. Chen and Spedding (1983) expressed the Butterworth (1975) correlation for turbulent–turbulent and laminar–laminar stratified flow in terms of the volumetric flow rate, Q, instead of the quality x,

$$\frac{H_G}{H_L} = K\left[\frac{Q_G}{Q_L}\right]^a \left[\frac{\rho_G}{\rho_L}\right]^b \left[\frac{\mu_G}{\mu_L}\right]^c \tag{A.26}$$

where K, a, b and c are dependent constants of flow regimen and the H_G/H_L range. For gas–liquid, turbulent–laminar stratified flow Chen and Spedding (1983) derived the following correlation:

$$\frac{H_G}{H_L} = \left[W_1 \frac{\rho_G}{\rho_L} \frac{q_G^{1.8}}{q_L} \frac{v_G^{0.2}}{v_L D^{0.8}}\right]^{1/\omega_1} \tag{A.27}$$

where $v = \mu/\rho$ is the kinematic viscosity, D the pipe diameter. For gas–liquid, laminar–turbulent flow Chen and Spedding (1983) proposed:

$$\frac{H_G}{H_L} = \left[W_2 \frac{\rho_G}{\rho_L} \frac{q_G}{q_L^{1.8}} \frac{v_G}{v_L^{0.2}} D^{0.8}\right]^{1/\omega_2} \tag{A.28}$$

where W_1, ω_1, W_2 and ω_2 are dependent constants of H_G/H_L.

Spedding and Chen (1984) applied the equation due to Armand (1946) Eq. (A.2) for bubble and slug types flows. The correlation proposed in annular flow for values of $H_G/H_L \geqslant 4$ is given by

$$\frac{H_G}{H_L} = 0.45\left[\frac{q_G}{q_L}\right]^{0.65} \tag{A.29}$$

Tandon et al. (1985) developed an analytical model to predict the void fraction in two-phase annular flow. This model is based in the Lockhart and Martinelli (1949) method.

$$H_G = 1 - 1.928 Re_L^{-0.315} [F(X_{TT})]^{-1} + 0.9293 Re_L^{-0.63} [F(X_{TT})]^{-2}, \quad 50 < Re_L < 1125 \tag{A.30}$$

$$H_G = 1 - 0.38 Re_L^{-0.088} [F(X_{TT})]^{-1} + 0.0361 Re_L^{-0.176} [F(X_{TT})]^{-2}, \quad Re_L > 1125 \tag{A.31}$$

$F(X_{TT})$ is a function of Lockhart–Martinelli X_{TT} parameter defined by

$$F(X_{TT}) = 0.15[X_{TT}^{-1} + 2.85 X_{TT}^{-0.476}] \tag{A.32}$$

where

$$X_{TT} = \left(\frac{\mu_L}{\mu_G}\right)^{0.1} \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_G}{\rho_L}\right)^{0.5} \tag{A.33}$$

Minami and Brill (1987) conduced an experimental study of two-phase flow to investigate liquid holdup in wet-gas pipelines and proposed a holdup correlation for horizontal flow,

$$H_L = 1 - \exp\left[-\left(\frac{\ln Z + 9.21}{8.7115}\right)^{4.3374}\right] \tag{A.34}$$

where Z is the Eaton et al. (1967) abscisse.

Hart et al. (1989) derived a holdup for horizontal gas–liquid pipe flow with a small liquid holdup ($H_L < 0.06$).

$$\frac{H_L}{H_G} = \frac{U_{SL}}{U_{SG}}\left[1 + 10.4 Re_{SL}^{0.363}\left(\frac{\rho_L}{\rho_G}\right)^{1/2}\right] \tag{A.35}$$

where $Re_{SL} = \mu_L U_{SL} D/\rho_L$ is the superficial Reynolds number of the liquid phase.

Abdul-Majeed (1996) simplified and improved the mechanistic model developed by Taitel and Dukler (1976) for estimating the holdup in horizontal two-phase flow. Abdul-Majeed (1996) demonstrated that the Taitel and Dukler (1976) model can be accurately represented by single explicit equations.

For turbulent flow:

$$H_L = \exp(-0.9304919 + 0.5285852R - 9.219634 \times 10^{-2}R^2 + 9.02418 \times 10^{-4}R^4) \tag{A.36}$$

For laminar flow:

$$H_L = \exp(-1.099924 + 0.6788495R - 0.1232191 \times 10^{-2}R^2 - 1.778653 \times 10^{-3}R^3 + 1.626819 \times 10^{-3}R^4) \tag{A.37}$$

where $R = \ln(X)$ and X is the Lockhart–Martinelli parameter defined as follows:

$$X^2 = \left[\frac{U_{SG}}{U_{SL}}\frac{\rho_G}{\rho_L}\frac{\mu_L}{\mu_G}\right]^m \frac{\rho_L U_{SL}^2}{\rho_G U_{SG}^2} \tag{A.38}$$

$m = 0.2$ for turbulent flow, whereas $m = 1$ for laminar flow.

Spedding et al. (1998) proposed a new relation between holdup, $Q_L/(Q_L + Q_G)$ and pipe diameter for $U_{SG} \geqslant 6$ m/s,

$$H_L = (3.5 + D)\left(\frac{Q_L}{Q_T}\right)^{0.7} \tag{A.39}$$

Gómez et al. (2000) developed a correlation to evaluate the liquid holdup in the slug body from horizontal to upward vertical flow,

$$H_L = \mathrm{Exp}(-0.45\theta_R - 2.48.10^{-6} Re_{LS}) 0 \leqslant \theta_R \leqslant 157 \tag{A.40}$$

where θ_R is the inclination angle in radians and $Re_{LS} = \rho_L U_M D/\mu_L$ is the liquid slug Reynolds number.

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